Pitch-Altitude Autopilot

Linear Control Systems Case Study

Rafael Marques Braga
University of Miami
Miami, FL
rxm403@maimi.edu

***Abstract*—This document will provide a solution of a linear control systems problem. It should be able to provide a control system for a pitch attitude autopilot that automatically adjusts a sudden weight shift of an aircraft if all passengers rush to the back of the plane.**

***Index Terms*—Pitch altitude autopilot, Linear control systems, lead controller, integral system.**

I. INTRODUCTION

The problem introduced in this paper is that "Golden Nugget Airlines has opened a free bar in the tail of their airplanes in an attempt to lure customers. In order to automatically adjust for the sudden weight shift due to passengers rushing to the bar when it first opens, the airline is mechanizing a pitch-attitude autopilot." [1] For that a linear control system is going to be designed and built using techniques such as basic feedback strategies, root locus analysis and design along with others.

The proposed control system of the for the pitch-attitude autopilot is the following:

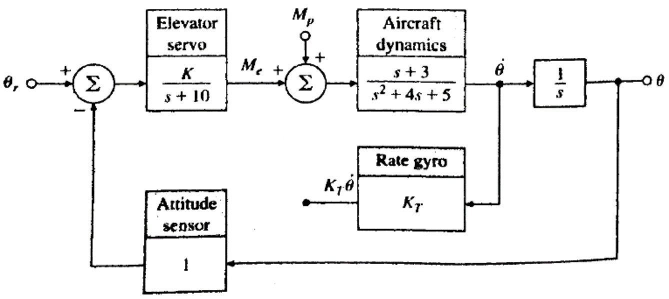


Figure 1 - Proposed Control System

When the input of the system, θ_r, is the desired angle of the airplane. M_p in the system is the step disturbance of the system, meaning the passenger movement from them going to the end of the plane. This disturbance has a given value of

$$M_p = \frac{M_0}{s}, \qquad M_0 \le 0.6$$

The output of the system, θ, is the final angle of the plane after the disturbance of the passengers and after being corrected by the control system, along with its components, the elevator servo (the controller, D(s)), the aircraft dynamics (the plant, G(s)), rate gyro and the attitude sensor (H(s)).

II. PROBLEM ANALYSIS

A. Finding the value of K to keep the steady state error in θ to be less than 0.02 radians (≈1°)

To find the value of K, in the elevator servo equation, to be less than 0.02 radians, first we need to find the open loop equation of the system. Since the error of the system will be introduced in M_p as the disturbance, the open loop equation that will have to be looked at is between the disturbance of the system, M_p, and the output, θ. This equation is found to be

$$\frac{\theta(s)}{M_p(s)} = \frac{G}{1+GDH}M_p = \frac{\frac{s+3}{s(s^2+4s+5)}}{1+\frac{(s+3)K}{(s+10)(s^2+4s+5)}} \cdot \frac{0.6}{s}$$

$$= \frac{(s+3)(s+10)}{s(s+10)(s^2+4s+5)+(s+3)K} \cdot \frac{0.6}{s}$$

Knowing the open loop equation of the system, it can be seen that it is a Type 1 system. Furthermore, using the final value theorem for a type 1 system and the equation for the steady state error being equal;

$$e_{ss} = \lim_{s\to 0} s\frac{\theta(s)}{M_p(s)} \le 0.02 = \frac{3*10*0.6}{3K} = 0.02$$

$$\therefore K = 300$$

After doing the calculations, for the steady state error in the system to be less than 0.02 radians, the value of K should be approximately equal to 300.

B. Drawing a root locus with respect to K

To find the root locus of the system with respect to K, the open loop function, L(s), of the characteristic equation, Δ, of the system should be used. This equivalent to the following:

$$L(s) = \frac{(s+3)K}{(s+10)(s^2+4s+5)}$$

Using the equation above and MATLAB the following root locus was found.

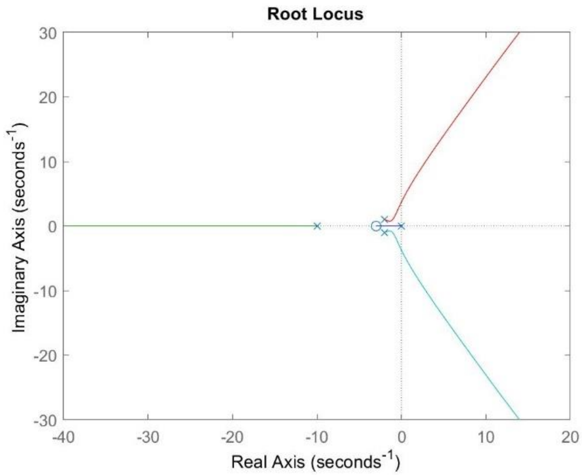


Figure 2 - Root locus of system

C. Finding the value of K for the system to be unstable

For the system to be unstable that means the poles are found in the right-hand side of the graph. For that to happen, using an estimate from the root locus found in *Part B,* K was found to be bigger than 133. This was the closest estimate possible when the pole was at -0.04+3.6i. Any K value higher than that, using MATLAB made the system unstable.

D. Supposing K is equal to 600, finding its unstable roots

If K is equal to 600, the characteristic equation, Δ, is used. The characteristic equation is equal to the open loop equation plus 1. This is equal to the following:

$$\Delta = 1 + L(s) = 1 + \frac{(s+3)K}{(s+10)(s^2+4s+5)}$$

$$= s^4 + 14s^3 + 45s^2 + 650s + 1800k$$

By plugging in that equation in MATLAB, and finding its roots, these can be found to be equal to -13.5014, 1.2183 ± 6.6284i and -2.9352.

As seen, it can be proved that when K = 600, it will yield to an unstable system since there are poles in the right hand side of the system, when the roots are equal to 1.2±6.62i.

E. Given a rate gyro, and with K=600, finding its best place to implement it.

If a rate gyro was given, the best place to implement it would be before the elevator servo, this way the controller would be able to take into account any change made to the information from the rate gyro and make the entire system stable using all the information it has available.

The new block diagram of the control system would now look like as shown in figure 2.

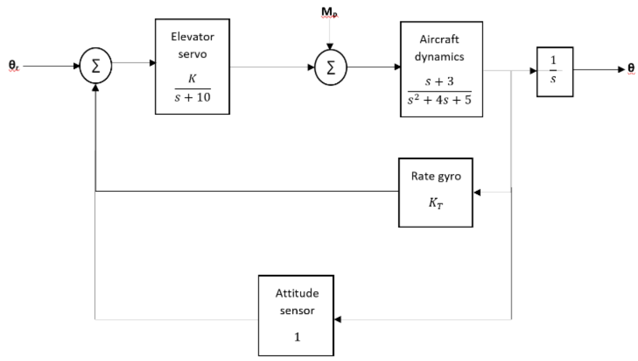


Figure 3 - New Block diagram (Part E)

F. Figuring out the root locus with respect to K_t

To find the root locus with respect to K_t, first the closed loop equation must be found. Using Mason's rule and the diagram seen in figure 3, the loop equation is found to be:

$$L(s) = \frac{(1-K_t)(Ks+3K)}{(s+10)(s^2+4s+5)s}$$

Using K = 600, the loop equation becomes

$$L(s) = \frac{(1-K_t)(600s+1800)}{(s+10)(s^2+4s+5)}$$

By inserting the equation in MATLAB, the following root locus is given

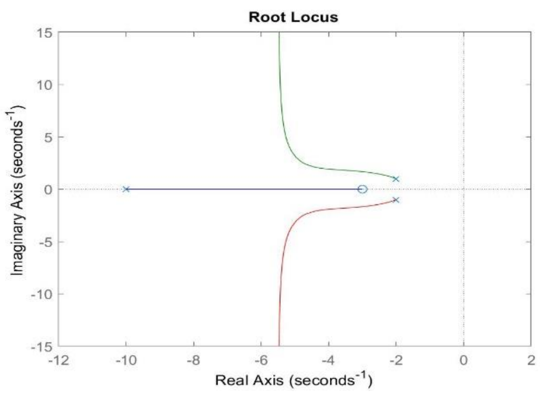
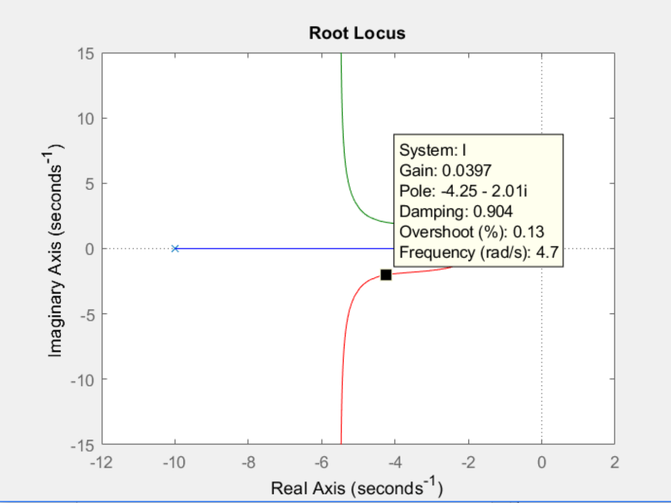


Figure 4 - New root locus of stable system

As seen in the root locus above the system is now stable with K = 600 and with any value of K_t when before it was an unstable system when K was equals to 600.

G. Finding the maximum damping factor of the complex roots

When examining the root locus from *part f* , it can be seen that the maximum damping factor would be of $\zeta = 0.904$ with the gain of 0.0397 as it can be seen in *figure 4. At this point the dominant pole is located at -4.25 ± 2.01i.*



H. Adding an integral term and an extra lead to the system

A lead compensator usually has the form of

$$G_c(s) = K_c\alpha\left(\frac{Ts+1}{\alpha Ts+1}\right)$$

With α being from 0 to 1. The advantages of a lead compensator are that it could improve the damping of the response of the system. Furthermore, for an integral controller, it would usually decrease the steady state error to 0, since the steady state error is equal to the reciprocal of the velocity constant and by adding a integrator to the system, it would increase the type of the system, giving a 0 error in the step response as well as in the impulse response which the steady state error was already 0. However, it would also make the damping decrease and it would make the settling time of the system longer.

III. Conclusion

In this paper, it was discussed on how a pitch pilot altitude system would be built using a set given of specifications. During the study of the problem, it was found a value for the system for it to be stable with a simple feedback loop, but after it was also seen on how adding a rate gyro would help the system to become more stable as the K value got higher. By adding the rate gyro, the system became stable with a much higher value of K (=600) when the highest value of K for it to be stable without the rate gyro was found to be 150 at the start of the problem. Through out the end of the problem it was seen that by adding a lead compensator to the system the damping response could be improved even further and by adding an integral controller the steady state error could even be reduced to 0.

IV. References

[1] G. F. Franklin, J. D. Powell, and A. Emami-Naemi, Feedback Control of Dynamic Systems. Upper Saddle River, NJ: Pearson, 2015.